Abstract

The Talenti theorem serves as a fundamental building block in the realm of Schwarz symmetrization for elliptic partial differential equations. Specifically, this theorem establishes a crucial result for weak solutions of the Laplacian with Dirichlet boundary conditions. It asserts that these solutions exhibit growth under symmetric decreasing rearrangement, leading to a pointwise inequality between them.

However, it's important to note that this result doesn't necessarily hold in the same manner for nonlocal cases. For the fractional Dirichlet problem:

$$\begin{cases} (-\Delta)^s u = f & \text{in } \ \Omega, \\ \\ u = 0 & \text{on } \ \mathbb{R}^N \setminus \Omega, \end{cases}$$

where Ω represents a bounded set within $\mathbb{R}^N$, and the function $f \in L^p(\Omega)$ is nonnegative, where $p > 1$ if $N = 1$, and $p > 2_s^*$ otherwise, the author's Vincenzo Ferone and Bruno Volzone has provided important insights, in their work "*Symmetrization for fractional elliptic problems: a direct approach*", including the identification of counterexamples that challenge the pointwise inequality between weak solutions.

Building on this research, our objective is to explore whether a pointwise inequality can be established in nonlocal scenarios, particularly when the region Ω is a ball with a radius of $r > 0$. This specific constraint allows us to use unique properties of the Martin kernel for the Green function in this scenario.