Goethe-Universität Frankfurt
Institut für Mathematik
Winter term 2020/21
2. Februar 2021

Algebra
Prof. Dr. Martin Möller
M.Sc. Jeonghoon So
M.Sc. Riccardo Zuffetti

Übungsblatt 12

No zoom lecture on Thursday. Please **read** (Satz von Artin-Schreier in Bosch p. 265) and **check** that as a consequent "solvable = solvable by radicals" also holds in positive characteristic.

Aufgabe 1 (4 Punkte)

Compute the elementary divisors (without the basis) of the following submodules:

(a) $M_1 = \langle (2,4,2), (8,2,18) \rangle_{\mathbb{Z}} \subseteq \mathbb{Z}^3$

(b) $M_2 = \langle (2x, x, x^3, 0, x^3 + x), (4x^2, 0, x^2 + x, x, x^4 + 2x^3) \rangle_{\mathbb{Q}[X]} \subseteq \mathbb{Q}[x]^5$

(c) $M_3 = \langle (x^2, x^9 + x^5 + x^4 + x^2, x^6 + x^2), (x^3 + x, f, x^7 + x^5 + x^3) \rangle_{\mathbb{Q}[x]} \subseteq \mathbb{Q}[x]^3$, where $f = x^{10} + x^8 + x^6 + x^5 + x^4 + x^3$.

Aufgabe 2 (4 Punkte)

Compute the elementary divisors and corresponding basis of the following submodules:

(a) $M_1 = \langle (1,0,-1), (4,3,-1), (0,9,3), (3,12,3) \rangle_{\mathbb{Z}} \subseteq \mathbb{Z}^3$

(b) $M_3 = \langle (x^2, x^9 + x^5 + x^4 + x^2, x^6 + x^2), (x^3 + x, f, x^7 + x^5 + x^3) \rangle_{\mathbb{Q}[x]} \subseteq \mathbb{Q}[x]^3$, where $f = x^{10} + x^8 + x^6 + x^5 + x^4 + x^3$.

Aufgabe 3 (4 Punkte)

Let R be an integral domain.

(a) Show that an ideal $I \subset R$ is a free R-module if and only if I is a principal ideal.

(b) Find a finitely generated torsion free $\mathbb{Z}[X]$-module which is not free.

Aufgabe 4 (4 Punkte)

Let R be a ring and R-modules M_1, M_2, M_3. Show that

(a) $M_1 \otimes_R M_2 \simeq M_2 \otimes_R M_1$

(b) $(M_1 \otimes_R M_2) \otimes_R M_3 \simeq M_1 \otimes_R (M_2 \otimes_R M_3)$.

In other words - the tensor product is symmetric and associative.

Please, upload your solutions on the Olat page of this course, by **14:00** on **Tuesday, 09.02.2021**.