Goethe-Universität Frankfurt
Institut für Mathematik
Sommersemester 2019
5. Juni 2019

Algebraische Geometrie II
Prof. Dr. Martin Möller
Dr. David Torres-Teigell
Dr. Matteo Costantini

Übungsblatt 4

Aufgabe 1 (4 Punkte)

Show that for a base-point free line bundle $\mathcal{L}$ on a compact complex manifold X it holds

$$\int_X \mathrm{c}_1(\mathcal{L})^n \geq 0.$$

Hint: Use the fact that pull-backs of vector bundles and Chern classes commute.

Aufgabe 2 (4 Punkte)

(a) Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be vector bundles over a compact complex manifold X. Show that the total Chern class satisfies

$$\mathrm{c}(\mathcal{E}_1 \oplus \mathcal{E}_2) = \mathrm{c}(\mathcal{E}_1) \cdot \mathrm{c}(\mathcal{E}_2)$$

and the Chern character satisfies

$$\mathrm{ch}(\mathcal{E}_1 \oplus \mathcal{E}_2) = \mathrm{ch}(\mathcal{E}_1) + \mathrm{ch}(\mathcal{E}_2), \quad \mathrm{ch}(\mathcal{E}_1 \otimes \mathcal{E}_2) = \mathrm{ch}(\mathcal{E}_1) \cdot \mathrm{ch}(\mathcal{E}_2).$$

(b) Let $\mathcal{E}$ be a vector bundle of rank k over a compact complex manifold X. Show that

$$\mathrm{c}_1(\mathcal{E}) = \mathrm{c}_1(\bigwedge^k \mathcal{E}).$$

Aufgabe 3 (4 Punkte)

(a) Show that there is an exact sequence of vector bundles

$$0 \to \mathcal{O}_{\mathbb{P}^n} \to \mathcal{O}_{\mathbb{P}^n}(1)^{\oplus n+1} \to \mathcal{T}_{\mathbb{P}^n} \to 0$$

where the surjection on the fiber over $[x] \in \mathbb{P}^n$ is given by the map

$$(\alpha_0, \ldots, \alpha_n) \mapsto \sum_{i=0}^{n} \alpha_i(x) \frac{\partial}{\partial X_i}.$$

(b) Compute all the Chern classes of the holomorphic tangent bundle $\mathcal{T}_{\mathbb{P}^n}$ of the complex projective space $\mathbb{P}^n$.

Aufgabe 4 (4 Punkte)

Let $\mathcal{E}$ be a vector bundle over a compact complex manifold X. Compute the first 4 degree terms of the Chern character $\mathrm{ch}(\mathcal{E})$ and the total Todd class $\mathrm{td}(\mathcal{E})$ in terms of the Chern classes $\mathrm{c}_i(\mathcal{E})$.

Abgabe: Zu Beginn der Übung um **14:15** Uhr am **Mittwoch, den 19 Juni**.